STATIONARY COMBUSTION MODES WITH ALLOWANCE FOR HEAT LOSSES

E. A. Chernova

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 4, pp. 86-91, 1967

An attempt is made to investigate the number of possible stationary combustion modes in a continuous-flow semi-infinite pipe with allowance for heat losses through the walls. Cases of a zero-order reaction in the reaction mixture or similarity of the concentration and temperature fields are considered. The equations are averaged with respect to the transverse coordinate η. Within the framework of these approximations it is found that the number of stationary combustion modes is determined by the roots θ_n of some function. The roots θ_{2k} correspond only to trivial unstable solutions. The roots θ_{2k-1} correspond to modes possible within broad regions of variation of the parameters characterizing the temperature of the mixture, the mixture feed rate, and the rate of heat removal. These regions intersect, forming zones where several stationary modes coexist. In these zones, apart from monotonic solutions there may also be solutions that initially make several oscillations. It is shown that the latter are obviously unstable and, in the last analysis, lead to one of the monotonic modes. The common case of not more than three roots is examined in detail.

If the heat release function can change sign, then a similar picture is also observed in the absence of heat losses through the walls (the roots θ_{2k-1} and θ_{2k} may change roles). In this case it is no longer necessary to average the equations with respect to η, since there will not be any corresponding derivatives.

We will consider a semi-infinite cylindrical pipe of radius r_0 into which a reaction mixture at temperature T_- is fed from the end at constant velocity w. The equations of stationary combustion in the pipe are taken in the form:

$$a\frac{\partial^2 u}{\partial \xi^2} + \frac{b}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial u}{\partial \eta}\right) - \frac{\partial u}{\partial \xi} + f(u) = 0\ ,$$

$$\xi = 0, \quad u = u_-; \qquad \eta = 0, \quad \partial u / \partial \eta = 0;$$

$$\eta = 1, \quad u = 0\ ,$$

$$u = \frac{(T - T_0)E}{RT_0^2}, \quad \xi = \frac{xF(T_0)E}{wRT_0^2}, \qquad \eta = \frac{r}{r_0},$$

$$f(u) = \frac{F(T)}{F(T_0)}, \quad a = \frac{\varkappa_1 F(T_0)E}{w^2 RT_0^2}, \quad b = \frac{\varkappa_2 RT_0^2}{r_0^2 F(T_0)E}. \qquad (1)$$

Here, T is temperature, T_0 is the temperature at the walls, E is the activation energy, R is the universal constant, x, r are the longitudinal and radial coordinates, $\varkappa_1$, $\varkappa_2$ = const are the effective thermal diffusivities in these directions, and $F(T)\rho c \geq 0$ is the heat release function (ρc = const is the volume specific heat of the mixture).

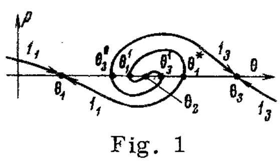

Fig. 1

Problem (1) corresponds to the case of similarity of the concentration C and temperature T fields, which is realized for $\varkappa_{1,2} = D_{1,2}$ (D is the diffusion coefficient) and similarity of the boundary conditions for C and T, which presupposes a continuous supply of active medium and the partial removal of combustion products through the walls. The case of a zero-order reaction also leads to problem (1). This can be used, for example, in connection with a small decrease in active medium in a sufficiently long pipe, a situation frequently encountered in chemical engineering. Neglecting the decrease in active medium majorizes the heat release function, which helps in estimating the region of conditions under which ignition, in this case undesirable, will not occur.

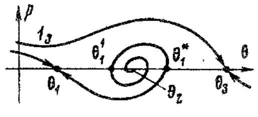

Fig. 2

In [1–4] the stationary combustion modes investigated in flat, cylindrical, and spherical vessels without through-flow are described in the cylindrical case by Eq. (1) without the first and third terms, and in the plane case by an equation of type (2) without the second and last terms. Stationary combustion in continuous-flow chambers has been mainly investigated, for example, in [5–7], without consideration of the heat loss through the walls, although the latter may sometimes play an important part. An attempt to study this effect was made in [8]. Equation (1) was investigated with the first term discarded (which is valid at small a) and the function $f(u)$ linearized. These simplifications heavily distort the qualitative picture; in particular, they do not allow consideration of the possibility of nonuniqueness of the solution.

We will investigate Eq. (1) by an integral method (averaging the equation with respect to η). In this case we represent $u(\xi, \eta)$, for example, in the form of a sum of powers of η not higher than the second with coefficients depending on ξ, i.e., with account for the boundary conditions $u(\xi, \eta) = \theta(\xi)(1 - \eta^2)$. Substituting an expression of this type into Eq. (1) and integrating the latter over the cross section of the pipe, we obtain

$$a\frac{d^2\theta}{d\xi^2} - \frac{d\theta}{d\xi} - \varphi(\theta) + \frac{\theta}{\delta} = 0\ ,$$

$$\delta = \frac{r_0^2 F(T_0)E}{8\varkappa_2 RT_0^2}, \qquad \varphi(\theta) = 4\int_0^1 f[\theta(1 - \eta^2)]\eta\, d\eta\ . \qquad (2)$$

The general form of Eq. (2) and all the subsequent analysis remain unchanged if we approximate $u(\xi, \eta)$ with respect to η by means of some other functions or, in general, make it independent of η (the heat transfer coefficient plays the part of $1/\delta$). The plane case also leads to Eq. (2).

We will find bounded solutions of Eq. (2). This excludes the indeterminacy consisting in the possibility of the presence of heat sources at infinity. For solutions taking the value θ_+ at infinity the boundary conditions can be taken in the form:

$$\xi = 0, \ \theta = \theta_-; \qquad \xi = \infty, \quad \theta = \theta_+\ . \qquad (3)$$

Hence it follows that $d\theta/d\xi = 0$, $d^2\theta/d^2\xi = 0$ at $\xi = \infty$ and, consequently, in accordance with (2), the values θ_+ must be roots of the function

$$\psi(\theta) = \varphi(\theta) - \theta / \delta\ . \qquad (4)$$

If $\psi(\theta)$ does not have roots, then there are no stationary combustion modes taking a definite value at infinity.

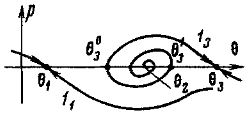

Fig. 3

Let $\psi(\theta)$ have N roots $0 \leq \theta_1 \leq \theta_2 \leq \ldots \leq \theta_N$ (the roots are nonnegative since $\psi(\theta) > 0$ and $\theta < 0$). Then problem (2), (3) can be reduced to the following N problems ($n = 1, 2, \ldots, N$):

$$\frac{dp}{d\theta} = \frac{1}{a} - \frac{\psi(\theta)}{ap}, \qquad \frac{d\theta}{d\xi} = p,$$

$$\xi = \infty, \quad \theta = \theta_n, \quad p = 0; \qquad \xi = 0, \quad \theta = \theta_-. \quad (5)$$

On the θp-plane the points $(\theta_n, 0)$ are singular points and, since there are no other singularities, they will be denoted by θ_n. In order to determine the form of θ_n we construct the characteristic equation with roots

$$\mu_{1,2} = {}^1/_2\,(1 \pm \sqrt{1 - \sigma_n}), \qquad \sigma_n = 4a\psi'(\theta_n). \quad (6)$$

We begin by assuming that $\psi(\theta)$ is continuous up to the first nonzero derivative and $|\psi'(\theta)| < \infty$. The function $\psi(\theta) > 0$ at $\theta < 0$. Therefore $\psi(\theta) > 0$ at $\theta < \theta_1$ and, consequently, in the case of simple roots $\psi'(\theta_1) < 0$, $\psi'(\theta_2) > 0, \ldots, \psi'(\theta_{2k-1}) < 0$, $\psi'(\theta_{2k}) > 0$, and so on. Hence, in accordance with (6), the point θ_{2k-1} ($k = 1, 2, \ldots$) will be a saddle point, and θ_{2k} a node at $\sigma_{2k} \leq 1$ and a focus at $\sigma_{2k} > 1$ (Figs. 1–3).

This sequence is also retained in the case of a multiple root $\theta_{n,i}$ ($\theta_n = \theta_{n+1} = \ldots = \theta_i$. In accordance with the analytic criteria given in [9], taking into account the sign of $\psi(\theta)$ at $n = 2k$, the left half θ_n of the point $\theta_{n,i}$ will be a node (the form of the curves near it is shown in Fig. 4), and at $n = 2k - 1$ a saddle point (Fig. 5). Similarly, the right halves θ_i of the points $\theta_{n,i}$ in Figs. 4, 5 correspond to the cases $i = 2j - 1$ (saddle point) and $i = 2j$ (node). If the forms of both halves of the point $\theta_{n,i}$ are the same (root of odd multiplicity), then it can be denoted by θ_n. The inclinations λ_1, λ_2 of curves passing through a saddle point and a node and the form of the curves near a focus in polar coordinates (B is an arbitrary constant) will be

$$\lambda_{1,2}(\theta_n) = \mu_{1,2}(\theta_n)\,a^{-1},$$

$$\rho = B \exp\left[-(\sigma_n - 1)^{-1/2}\,\omega\right]. \quad (7)$$

If we assume that motion along the curves in the θp-plane proceeds in the direction of an increase in ξ ($d\theta/p = d\xi > 0$), then any curve traveling from the straight line $\theta = \theta_-$ and arriving at the point θ_n will ensure the solution of problem (5), and conversely. In fact, if we take the curvilinear integral

$$\int_{(\theta_-,\,\theta)} \frac{d\theta}{p} = \xi(\theta)$$

along such a curve, then $\xi(\theta_-) = 0$, $\xi(\theta_n) = \infty$ (at the point θ_n the integral converges), and $\xi(\theta)$, increasing monotonically along the indicated curve, uniquely defines the unknown function $\theta(\xi)$. The converse is obvious.

In the upper half-plane, where $p > 0$, motion along the curves takes place from left to right, and in the lower half-plane from right to left. Therefore at points θ_{2k} (Figs. 1–3) the curves converge, and there are no solutions with $\theta_+ = \theta_{2k}$ (except for the trivial solution $\theta \equiv \theta_{2k}$ at $\theta_- = \theta_{2k}$, which may be assumed unstable).

The curve 1_{2k-1} with inclination $\lambda_2(\theta_{2k-1}) \leq 0$ arrives at point $\theta_{2k-1,2j}$ (Fig. 5) from the left, at point $\theta_{2s,2k-1}$ (Fig. 4) from the right, and at point θ_{2k-1} (Figs. 1–5) from both sides. It remains to establish the values of θ_- for which these curves give a solution.

Let $\psi(\theta)$ have the roots $\theta_1 < \theta_2 < \theta_3$. From (5), taking into account the sign of $\psi(\theta)$, there follows

$$\left|\frac{dp}{d\theta}\right|_{p \neq 0} < \infty,$$

$$\left(\frac{dp}{d\theta}\right)_{p=\pm 0} = \begin{cases} \mp\infty & (\theta < \theta_1,\ \theta_2 < \theta < \theta_3) \\ \pm\infty & (\theta_1 < \theta < \theta_2,\ \theta > \theta_3) \end{cases}. \quad (8)$$

If we proceed from point θ_1 along the curve 1_1 against the motion, then in accordance with (8), its left branch cannot turn back, intersect the θ-axis or have a vertical asymptote, but proceeds always to the left (Figs. 1–3), ensuring a solution with $\theta_+ = \theta_1$ (first mode, for all $\theta_- \leq \theta_1$ (only $\theta_- > -E/RT_0$ has physical significance). However, its right branch, proceeding to the right, either intersects the θ-axis at the point $\theta_1^* \in [\theta_2, \theta_3]$ (Figs. 1, 2) or proceeds further (Fig. 3) to infinity ($\theta_1^* = \infty$). If $\theta_1^* = \theta_2$ or $\theta_1^* = \theta_3$, then it is possible to proceed only to the point θ_1^*, since reaching (leaving) θ_n corresponds to infinite changes in ξ.

If $\theta_2 < \theta_1^* < \theta_3$, then from θ_1^* curve 1_1, in accordance with (8), proceeds to the left into the upper half-plane, approaches the straight line $\theta = \theta_2$ and intersects the θ-axis at the point $\theta_1 < \theta_1^1 \leq \theta_2$, since, in accordance with (5)–(7),

$$(dp\,/\,d\theta)_{p>0} = -(dp\,/\,d\theta)_{p<0} + 2\,/\,a. \quad (9)$$

At θ_1^1 curve 1_1 again turns and proceeds to the right in the lower half-plane, intersecting the θ-axis at $\theta_2 \leq \theta < \theta_1^*$, and so on. As a result, it arrives at the point θ_2, making an infinite number of loops at $\sigma_2 > 1$ (Fig. 2) and a finite number at $\sigma_2 \leq 1$ (Fig. 1). Thus, the right branch of curve 1_1 gives a solution with $\theta_+ = \theta_1$ for $\theta_1 \leq \theta_- \leq \theta_1^*$ (at $\theta_1^* = \theta_2$ or $\theta_1^* = \theta_3$ for $\theta_1 \leq \theta_- < \theta_1^*$), while at $\theta_1^1 \leq \theta_- \leq \theta_1^*$, $\theta_2 < \theta_1^* < \theta_3$ to the monotonic solution there must be added solutions (at $\theta_+ = \theta_2$, $\sigma_2 > 1$ there are infinitely many) which first make a finite number of oscillations about θ_2.

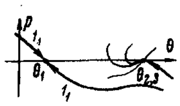

Fig. 4

Similarly, curve 1_3 gives a solution with $\theta_+ = \theta_3$ (third mode) for $\theta_- \geq \theta_3^0$ (at $\theta_3^0 = \theta_2$ or $\theta_3^0 = \theta_1$ for $\theta_- > \theta_3^0$), where $\theta_3^0 \in [\theta_1, \theta_2]$ (Figs. 1, 3) or $\theta_3^0 = -\infty$ (Fig. 2). At $\theta_3^0 \leq \theta_- \leq \theta_3^1$, $\theta_1 > \theta_3^0 < \theta_2$ there will be solutions that oscillate for a certain time about θ_2.

An analysis shows that at $\theta_- < \theta_3^0$ only the first mode ($\theta_+ = \theta_1$) can exist, and at $\theta_1 > \theta_1^*$ only the third ($\theta_+ = \theta_3$). At $\theta_- \in [\theta_3^0, \theta_1^*]$ both these modes are possible, and in this case, together with the monotonic, there may also be solutions that first oscillate about

θ_2, while at $\theta_- = \theta_2$ there is also an unstable second mode ($\theta = \theta_2$). The point $\theta_1{}^*$, which either lies on the interval $[\theta_2, \theta_3]$ or goes to infinity, will be the maximum temperature to which it is possible to heat the mixture in order to realize the first mode, which is usually slow flameless combustion. As θ_- increases, only the third high-temperature mode will be possible. Therefore θ^* may be called the ignition temperature, and $\theta_3{}^0$, by analogy, the quenching temperature ($\theta_1 \leq \theta_3{}^0 \leq \theta_2$, or $\theta_3{}^0 = -\infty$).

We will estimate the values $\theta_1{}^*$, $\theta_3{}^0$. In accordance with (5)–(7),

$$\left(\frac{dp}{d\theta}\right)_{p<0} > \frac{1}{a} + \frac{m}{ap}, \quad \left(\frac{dp}{d\theta}\right)_{p>0} > \frac{1}{a},$$

$$m = -\min \psi(\theta) \quad (\theta_1 \leqslant \theta \leqslant \theta_2),$$

$$\left(\frac{dp}{d\theta}\right)_{p<0} > \frac{1}{a}, \quad \left(\frac{dp}{d\theta}\right)_{p>0} > \frac{1}{a} - \frac{M}{ap},$$

$$M = \max \psi(\theta) \quad (\theta_2 \leqslant \theta \leqslant \theta_3).$$

Integration of these inequalities gives the following estimates valid on the interval $[\theta_1, \theta_3]$:

$$\frac{\theta_1{}^* - \theta_2}{am} - \ln\left[1 - \frac{\theta_1{}^* - \theta_2}{am}\right] < \frac{\theta_2 - \theta_1}{am},$$

$$\frac{\theta_2 - \theta_3{}^\circ}{aM} - \ln\left[1 - \frac{\theta_2 - \theta_3{}^\circ}{aM}\right] < \frac{\theta_3 - \theta_2}{aM}. \quad (10)$$

From the inequality $dp/d\theta > -\psi(\theta)/ap$ it follows that $\theta_1{}^* < \Theta_1{}^*$, and $\theta_3{}^0 > \Theta_3{}^0$, where $\Theta_1{}^*$ and $\Theta_3{}^0$ are roots of the functions

$$\int_{\theta_1}^{\theta} \psi(\theta)\, d\theta, \quad \int_{\theta}^{\theta_3} \psi(\theta)\, d\theta,$$

adjacent to θ_1 and θ_3, respectively. Hence it follows that

$$0 \leqslant \theta_1{}^* - \theta_2 \leqslant \min\left[\Theta_1{}^* - \theta_2,\ am,\ \sqrt{2am(\theta_2 - \theta_1)}\right],$$

$$0 \leqslant \theta_2 - \theta_3{}^\circ \leqslant \min\left[\theta_2 - \Theta_3{}^\circ,\ aM,\ \sqrt{2aM(\theta_3 - \theta_2)}\right]. \quad (11)$$

The last two estimates in the brackets are suitable as long as they do not exceed $\theta_3 - \theta_2$ and $\theta_2 - \theta_1$, respectively. If Q = 0, where

$$Q = \int_{\theta_1}^{\theta_3} \psi(\theta)\, d\theta,$$

then $\Theta_1{}^* = \theta_3$, $\Theta_3{}^0 = \theta_1$ and, consequently, $\theta_1{}^* < \theta_3$, $\theta_3{}^0 > \theta_1$; if Q > 0, then, keeping in mind the signs of the integrand function ψ, we obtain $\Theta_1{}^* < \theta_3$, $\Theta_3{}^0 = -\infty$, i.e., $\theta_1{}^* < \theta_3$; if Q < 0, then $\Theta_3{}^0 > \theta_1$, $\Theta_1{}^* = \infty$ and $\theta_3{}^0 > \theta_1$. Thus, when Q = 0 both critical temperatures ($\theta_1{}^*$ and $\theta_3{}^0$), at Q > 0 the ignition temperature, and at Q < 0 the quenching temperature exist at any a (Q increases with increase in δ).

If a decreases, then, in accordance with (11), for any Q there is an a^* (for Q = 0 this is $a = \infty$), for which $\theta_1{}^* < \theta_3$, $\theta_3{}^0 > \theta_1$. With further decrease in a (for example, increase in the mixture feed rate w) $\theta_1{}^*$ and $\theta_3{}^0$ approach θ_2, contracting the region of nonuniqueness of the solutions; the amplitude and number of the oscillations in the oscillatory solutions, in accordance with (9), decrease, θ_2 becomes a node, and a moment arrives at which $\theta_1{}^* = \theta_3{}^0 = \theta_2$.

As a increases, the points $\theta_1{}^*$ and $\theta_3{}^0$ converge, respectively approaching $\Theta_1{}^*$ and $\Theta_3{}^0$, so that at $a > a^*$ for Q > 0 the third mode and for Q < 0 the first mode become possible at any θ_-, and the quenching (ignition) temperature ceases to exist. As a increases, the point θ_2 becomes a focus, and at $a = \infty$ (w = 0) a center. The number of oscillatory solutions and the number of oscillations about θ_2 increase, and at $a = \infty$ they degenerate into an infinite set of solutions executing periodic oscillations about θ_2 with an amplitude from 0 to min ($\Theta_1{}^*$, θ_3) — max ($\Theta_3{}^0$, θ_1) and not taking a definite value at x = ∞. (At $a = \infty$, taking $\xi = x/r_0$, we obtain $dp / d\theta = -\psi(\theta) / bp$ and $\theta_1{}^* = \Theta_1{}^*$, $\theta_3{}^0 = \Theta_3{}^0$.)

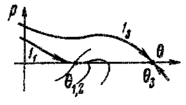

Fig. 5

In the case of a small increase (decrease) in temperature when the oscillatory solution intersects the θ from the right (left) there may be a transition to an oscillation of greater amplitude (at $a < \infty$ to a solution making a smaller number of oscillations), and it may therefore be expected that the oscillatory solutions will be unstable and, in the last analysis, lead to one of the two monotonic modes (with $\theta_+ = \theta_1$ or $\theta_+ = \theta_3$).

If δ decreases, then θ_1 and θ_3 decrease, while θ_2, $\theta_1{}^*$, and $\theta_3{}^0$ increase, so that at $\delta < \delta_1(a)$ we have $\theta_1{}^* = \infty$, at $\delta = \delta^0$ the points θ_2 and θ_3 coincide forming the point $\theta_{2,3} = \min_\delta \theta_3$ (Fig. 4). At $\delta < \delta^0$ the point $\theta_{2,3}$ disappears altogether, and for all θ_- only the first flameless mode will exist. Therefore $\theta_{2,3}$ may be called the extinction temperature. (It is assumed that the function $\psi(\theta)$ does not have roots other than those considered.)

As δ increases the opposite picture is observed. At $\delta > \delta_3(a)$ we have $\theta_3{}^0 = -\infty$, while $\delta = \delta^*$ corresponds to $\max_\delta \theta_1 = \theta_{1,2}$ (Fig. 5). The point $\theta_{1,2}$ is called the combustion temperature, since at $\delta > \delta^*$ there is no such temperature and the first mode ceases to exist.

The critical values δ^0, δ^* and $\theta_{2,3}$, $\theta_{1,2}$ are determined from the equations

$$\psi(\theta, \delta) = 0, \quad d\psi / d\theta = 0. \quad (12)$$

If $K = \lim\,[\theta^{-1}\varphi(\theta)] \neq 0$ as $\theta \to \infty$, which may occur in the case of a zero-order reaction, then at $\delta > 1/K$ θ_3 disappears and steady-state modes will be impossible at $\delta > \max(1/K, \delta^*)$ for any θ_- and a, and at $1/K < \delta \leq \delta^*$ (if $1/K < \delta^*$) only for $\theta_- > \theta_1{}^*$ (onset of explosion).

Thus, the low-temperature flameless mode corresponding to θ_1 is possible at $\delta < \delta^0$ and at $\delta^0 \leq \delta \leq \delta^*$, $\theta_- \leq \theta_1{}^*(a, \delta)$; the high-temperature mode corresponding to θ_3 is possible at $\delta^* < \delta < 1/K$ (if $\delta^* < 1/K$) and at $\delta^0 \leq \delta < \min(\delta^*, 1/K)$, $\theta \geq \theta_3{}^0(a, \delta)$ (in the case of similarity of the C and T fields it is possible at $\delta > \delta^*$ and at $\delta^0 \leq \delta \leq \delta^*$, $\theta_- \geq \theta_3{}^0$). In these regions outside their zone of intersection the steady-state modes are unique. In that zone itself, i.e., at $\delta^0 \leq \delta < \min(\delta^*, 1/K)$, $\theta_3{}^0 < \theta_- < \theta_1{}^*$ both the above-mentioned modes exist, and, together with monotonic solutions, there may also be solutions that oscillate for a certain time about θ_2, which clearly will be unstable. At $\delta^0 < \delta < \delta^*$, $\theta_- = \theta_2$ an unstable mode corresponding to θ_2 is possible.

The above analysis is easily extended to the case $N > 3$.

It should be noted that, apart from the basic dependence on T_0, given by δ and a, the function $\varphi(\theta)$ in (4) and, consequently, all the critical quantities associated with it may have an additional (usually very weak) dependence on T_0 (if $F(T)$ has a rate constant expressed by the Arrhenius law in the Zel'dovich-Frank-Kamenetskii approximation, then there is no such dependence). However, if it is considerable, it is better to isolate T_0 as an independent parameter, introducing the dimensionless variables in some other way.

The solutions with $\theta = \theta_+$ at $\xi = \infty$ exhaust all the bounded solutions of Eq. (2), since, in accordance with (8), (9), at $a < \infty$ there are no closed curves on the θp-plane.

Let the function ψ or its derivatives have a discontinuity of the first kind at the point $\theta = A$. The picture previously obtained is basically preserved, if at $A = \theta_n$, $\operatorname{sign}\psi(A - \varepsilon) = \operatorname{sign}\psi(A + \varepsilon)$ (ε is arbitrarily small) A is assumed to be a root of even and at $\operatorname{sign}\psi(A - \varepsilon) = -\operatorname{sign}\psi(A + \varepsilon)$ odd multiplicity (in the case $\psi(A \pm 0) \neq 0$ the curves approaching the point A from the right (left) are not solutions).

If $\psi(\theta) \equiv 0$, $\theta_n \leq \theta \leq \theta_\nu$, then the interval $[\theta_n, \theta_\nu]$ will be a singular solution, motion along which is impossible ($p = 0$). From any point on this interval curves depart with an inclination $1/a > 0$. Therefore for $\theta_n < \theta_+ < \theta_\nu$ only the trivial solutions $\theta \equiv \theta_-$ at $\theta_n < \theta_- < \theta_\nu$ are possible. The points θ_n and θ_ν may be regarded as the two halves of a single point $\theta_{n,\nu}$ that is subject to the general rule. Thus, at $\psi(\theta_n - \varepsilon) > 0$ ($n = 2k - 1$), $\psi(\theta_n - 0) = 0$ the curve $1_n(1_\nu)$ arrives at the point θ_n from the left and at $\psi(\theta_\nu + \varepsilon) < 0$ ($\nu = 2j - 1$), $\psi(\theta_\nu + 0) = 0$ at the point θ_ν from the right, giving a solution with $\theta_+ = \theta_n(\theta_\nu)$ for $\theta_- \in [\theta_n^0, \theta_n]$ and $\theta_- \in [\theta_\nu, \theta_\nu^*]$, respectively. Otherwise, there are no solutions with $\theta_+ = \theta_n(\theta_\nu)$, apart from the trivial solution.

If $\varphi(\theta) \equiv 0$ ($\psi = -\theta/\delta$), $\theta \leq \beta$, then at $\beta > 0$ or $\beta = 0$, $\psi(\varepsilon) < 0$, $\psi(+0) = 0$ for $\theta_- < \theta_3^0$ there will always be total damping of the process ($\theta_+ = \theta_i = 0$); for $\theta_- > \theta_1^*$ it is impossible, while for $\theta_- \in [\theta_3^0, \theta_1^*]$, together with total damping, there exists a mode with $\theta_+ = \theta_3$; at $\beta = 0$, $\psi(\varepsilon) > 0$ the mode with $\theta_+ = \theta_{1,2} = 0$ is possible only at $\theta_- \leq 0$ (for any δ), at $\beta < 0$ it never exists.

REFERENCES

1. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics [in Russian], Izd-vo AN SSSR, 1947.

2. I. M. Gel'fand, "Some problems of quasi-linear equations," Uspekhi matem. nauk, vol. 14, no. 2, 1959.

3. A. G. Istratov and V. B. Librovich, "Stability of solutions in the steady-state theory of thermal explosion," PMM, vol. 23, no. 2, 1963.

4. S. A. Kaganov, "Stationary theory of thermal autoignition," PMTF, no. 1, 1963.

5. R. M. Zaidel and Ya. B. Zel'dovich, "Possible regimes of steady-state combustion," PMTF, no. 4, 1962.

6. A. G. Merzhanov and A. K. Filonenko, "Thermal autoignition of a homogeneous gas mixture in a flow," DAN SSSR, vol. 152, no. 1, 1963.

7. E. A. Chernova, "Stationary combustion," PMTF [Journal of Applied Mechanics and Technical Physics], no. 6, 1966.

8. S. A. Kaganov, "Flame propagation processes," PMTF [Journal of Applied Mechanics and Technical Physics], no. 4, 1965.

9. V. V. Nemytskii and V. V. Stepanov, Qualitative Theory of Differential Equations [in Russian], Gostekhizdat, 1949.

9 January 1967 Moscow